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An evaluation of the performance of phenomenological models for predicting pressure gradient during gas–liquid flow in horizontal pipelines

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Abstract

A review of the literature has revealed the lack of a formal analysis of the performance of phenomenological methods for the prediction of pressure drop during gas–liquid two-phase flow. A number of models describing various horizontal flow patterns are evaluated against a large experimental data bank using a logarithmic statistical analysis technique. It is shown that the precision and accuracy of phenomenological models are equal to those of empirical methods, while the probability density function is less sensitive to changes in fluid system. Two composite methods consisting only of phenomenological models are defined. The second of these methods generates predictions that are more reliable over the entire range of the data bank than all empirical methods included in the study. It achieves a 10% reduction in scatter ($S_{\ln}$) compared to the best empirical methods. This method is also shown to be less sensitive to changes in fluid system. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Pressure drop; Two-phase flow; Phenomenological models; Statistical analysis

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1. Introduction

The prediction of design parameters such as pressure drop during gas–liquid flow is achieved by one of the three approaches: empirical correlations, analytical models or phenomenological models.

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While the empirical approach requires a minimum of knowledge of the system characteristics, it is limited by the range of data available for correlation construction. Dimensional analysis of two-phase systems is not possible and no single correlation is capable of acceptable accuracy in a generic sense, testimony to the non-linearity of gas–liquid systems.

Previous evaluations of prediction methods have concentrated on empirical correlations. Their recommendations, summarised in Table 1, assume a causal link between the accuracy of empirical methods and the flow pattern, despite interfacial structure not necessarily being considered in the original correlation development.

In the phenomenological approach, models are developed based on the interfacial structure. A complete picture of the flow is obtained through the inclusion of phenomenon specific information such as slug frequency and interfacial shear stress. Modelling on a theoretical basis intends to reduce the dependence on empirical data, although some empiricism is still required. The simultaneous prediction of pressure gradient, void fraction and, if necessary, the heat transfer coefficient means the approach is now preferred. For design purposes these models are often brought together within a framework provided by a flow pattern map.

Although the advantages of such models are clear, no quantitative analysis has been published in the literature to validate their accuracy. A lack of trust in design procedures using phenomenological technology has been expressed (Jenkins, 1995), and can be related to the lack of validation across a wide range of conditions. This paper provides an evaluation of phenomenological models of horizontal gas–liquid flow against 7000 experimental pressure drop data, and compares their performance to those of empirical methods.

2. Statistical methods

Various statistical methods may be used to define an error distribution. The majority of these assume a unimodal Gaussian probability density function (PDF) and represent it via a mean value and spread about that mean. The general form of a PDF is given in Eq. (1), where f is the relative frequency, μ the mean and σ the standard deviation.

It has been shown (Tribbe, 1998) that the best approximation of reality is given by the

Table 1
Recommendations for flow pattern based composite prediction methods from previous evaluative studies in horizontal pipe flow

	Stratified	Wavy	Slug	Plug	Annular	Bubbly
Dukler et al. (1964)	Lockhart–Martinelli	Lockhart–Martinelli	Lockhart–Martinelli	Chenoweth and Martin	Lockhart–Martinelli	Lockhart–Martinelli
Anderson and Russell (1965)	Dukler	Dukler	Dukler	Chenoweth and Martin	Dukler	Dukler
Mandhane et al. (1977)	Agrawal et al.	Dukler	Dukler	Chenoweth and Martin	Chenoweth and Martin	Dukler
Ferguson and Spedding (1995)	Hanratty	Hanratty	No recommendation	Olujic	Olujic	No recommendation

distribution of logarithmic ratios. This is defined by Eqs. (2)–(4), in which N is the number of data. The distribution is defined by the mean and scatter of logarithmic ratios, $\bar{X}_{\ln}$ and $S_{\ln}$. An example of the comparison between true and representative distributions is given in Fig. 1; the errors are those that are associated with predictions of cryogenics data made by the Olujic (1985) correlation.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \tag{1}$$

The average of logarithmic ratios;

$$\bar{X}_{\ln} = \exp\left(\frac{1}{N}\sum_{i=1}^{N} X_{i,\ln}\right) - 1 \tag{2}$$

Scatter of logarithmic ratios;

$$S_{\ln} = \exp\sqrt{\frac{\sum_{i=1}^{N} X_{i,\ln}^2}{N - d_f - 1}} - 1 \tag{3}$$

where d_f = degrees of freedom and

$$X_{i,\ln} = \ln(R) = \ln\left[\frac{\left(\frac{dP}{dL}\right)_{i,\mathrm{EXP}}}{\left(\frac{dP}{dL}\right)_{i,\mathrm{CALC}}}\right] \tag{4}$$

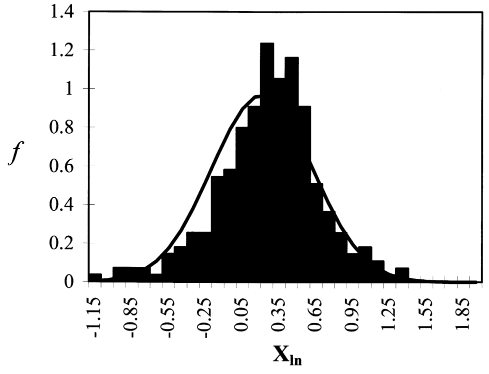


Fig. 1. Error distribution and Gaussian representation of predictions of cryogenics data by the Olujic (1985) method.

3. Data bank

The TVT-Dukler data bank includes almost 7000 experimentally derived data points of pressure drop during gas–liquid flow in horizontal pipes. Six fluid systems are represented in the data bank: air–water, steam–water, air–oil, air–aqueous CMC solutions, refrigerants and cryogenics. The non-Newtonian fluid data are not considered in this paper. A complete description of the contents of the TVT-Dukler data bank is given by Tribbe (1998).

Table 2
Prediction methods used in evaluation

Author(s)	Year
Empirical methods	
Bandel	1973
Beattie and Whalley	1982
Friedel	1979
Lockhart and Martinelli	1949
Müller-Steinhagen and Heck	1986
Olujic	1985
Phenomenological Methods	
Stratified flow models	
Agrawal et al.	1973
Baker and Gravestock	1987
Hanratty	1987
Hart et al.	
Hashizume et al.	1985
Johannessen	1972
Persen	1984
Russel et al.	1974
Taitel and Dukler	1976a
Taitel et al.	1989
Bubbly/dispersed bubble flow models	
Nicholson et al.	1978
Taitel et al.	1989
Slug flow models	
Dukler and Hubbard	1975
Nicholson et al.	1978
Taitel et al.	1989
Annular flow models	
Hart et al.	1989
Hashizume et al.	1985
Taitel et al.	1989
Composite phenomenological models	
Hashizume et al.	1985
Mukherjee and Brill	1985
Taitel et al.	1989

4. Correlations and models analysed

A total of 35 methods were analysed. Twenty empirical methods were evaluated, although only the results of six selected correlations are presented in this paper. The complete set of results is given by Tribbe (1998). The methods discussed here are listed in Table 2.

Each of the empirical methods reported in this paper is constructed in a distinctly different manner. Highly empirical methods are presented by Friedel (1979) and Müller-Steinhagen and Heck (1986). The former was constructed using a large data bank of two-phase pressure drop data, while the latter simply reflects aspects of symmetry observed in the relationship between the two-phase multiplier and flow quality. Other methods have evolved out of the homogeneous model, in which suitably defined physical properties of homogeneous fluid are used in conjunction with well known single-phase hydraulic relationships. One such method is proposed by Beattie and Whalley (1982), where the mixture viscosity is defined by the superposition of annular and bubble viscosities. The Lockhart and Martinelli (1949) method is based on the heterogeneous model and provides a correlation between the two-phase multiplier and the dimensionless parameter, X, defined as the ratio of superficial liquid and gas pressure drops. The underlying model assumes two separate phases flowing simultaneously, and the relationship between these two parameters is indicative of the interfacial interaction.

In recognition that a single method would not suffice, many attempts were made to combine the best of separate methods. The method described by Olujic (1985) is one of the number that sought to improve prediction in this way. Bandel (1973) related the observed change in pressure drop characteristics to the transition between stratified and annular flow regimes. Unlike Olujic's method, this structural variation is included within a common framework derived from a two-fluid analysis.

Phenomenological methods have become the desired approach due to their independence from fluid system and simultaneous prediction of various two-phase phenomena. The five regimes considered here are those included in the flow pattern map of Taitel and Dukler (1976a), i.e. stratified, wavy, annular, intermittent and dispersed bubble flow.

Smooth and wavy stratified flows are predicted using the same models. The majority of methods assume circular pipe geometry, one-dimensional (1D) velocity profiles in both phases and an interfacial friction factor relationship. Early models treated the interface as a free surface with respect to the liquid phase and as a stationary surface to the gas phase, neglecting interfacial friction (Johannessen, 1972; Russell et al., 1974). This parameter was included in subsequent models, usually as a function of the gas phase velocity (Hanratty, 1987; Persen, 1984; Taitel and Dukler, 1976b) or as a constant value (Taitel et al., 1989). Alternatively, the assumption of 1D flow between two flat plates can be made (Hashizume et al., 1985). A 2D velocity profile was assumed for the liquid phase by Agrawal et al. (1973). Interestingly, their chosen interfacial shear stress correlation is based on gas velocity only and is, therefore, largely independent of any radial variation in liquid velocity.

The model of Hart et al. (1989) is unusual in that it represents a flow transition. They describe the transition between stratified and annular flows by a liquid film that only wets a fraction of the pipe circumference. Although the model is specified for use at low void fractions, it has been evaluated beyond the constraint in this analysis.

The annular flow model of Hashizume et al. (1985) is based on an analogy with the falling film, using mixing length theory to define 1D shear stress and velocity profiles. The other models evaluated (Hart et al., 1989; Taitel et al., 1989) describe the interfacial friction factor of this film as an equivalent sand roughness, respectively, $2.3\delta/d$ and $4\delta/d$ (after Wallis, 1970). It is worth noting that Taitel et al. (1989) use Wallis' correlation with respect to the slip velocity and not the gas velocity as originally proposed, with the potential result of underpredicting pressure drop. None of the models incorporate entrainment or radial film thickness variation.

Intermittent flow models are based on the unit cell concept introduced by Dukler and Hubbard (1975), in which the intermittent flow structure is described as a repeating unit made up of a liquid slug and a trailing film. The model considers the passage of a slug to be a continual process of picking up and shedding liquid. Modifications were made by Nicholson et al. (1978) and a simplified model was presented by Taitel et al. (1989). Empirical information is used to describe intermittent flow phenomena such as slug frequency, hold-up and length as well as interfacial friction factors. Following a simplification proposed by Nicholson et al. (1978), an assumed slug length of 30 pipe diameters appears to have become widely accepted.

Nicholson et al. (1978) extended their model to incorporate the transition to dispersed bubble flow, considering it to be a homogeneous fluid. Taitel et al. (1989) modelled dispersed bubble flow as a Taylor bubble flow with no drift velocity, rather unlike the treatment made in the flow pattern map of Taitel and Dukler (1976a).

The composite models evaluated include the models of Hashizume et al. (1985) and Taitel et al. (1989), as described previously. In addition is the composite model by Mukherjee and Brill (1985), which uses a combination of empirical and mechanistic modelling to describe various regimes flowing in pipes of varying inclination angle.

5. Results and discussion

As phenomenological models are defined by the interfacial structure they represent, discussions of the results are grouped into the five flow pattern categories defined by Taitel and Dukler (1976a). The same models evaluated for stratified smooth flow are also evaluated for stratified wavy flow. The results of the flow pattern specific statistical analysis for phenomenological models are listed in Table 3.

This is followed by an analysis of the generic prediction capabilities of composite models recommended from the flow pattern specific analyses. Additional qualitative information regarding the relationship between error and the flow pattern map is also employed in the model selection procedure. The associated error statistics are listed in Table 5.

5.1. Stratified smooth flow

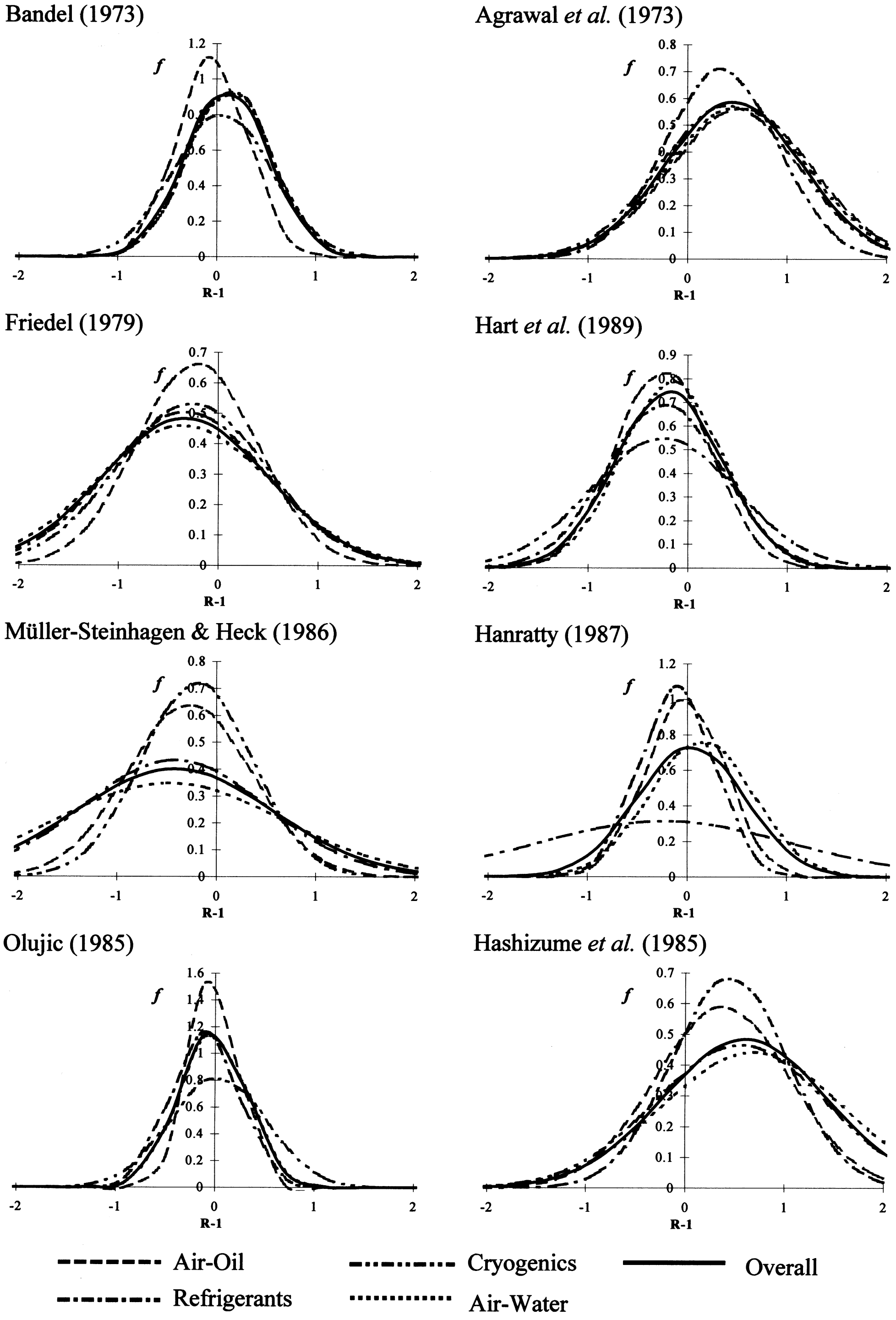
Pressure drops experienced during flow in this regime are small, and even small absolute errors appear to be large on a relative scale. Accurate predictions appear difficult to make, and precision is limited.

In this investigation, particularly precise predictions are made using the stratified flow model of Agrawal et al. (1973), a finding in agreement with that of Mandhane et al. (1977). The model also has the considerable advantage in that its performance is only weakly influenced by variations in fluid system.

Apart from the Agrawal et al. (1973) model, the precision of phenomenological models is poor. Generally, more reliable predictions are provided by empirical methods, of which the Bandel (1973) correlation is the most precise. The rather unsatisfactory performance of the

Table 3
PDF parameters for flow pattern specific prediction methods

Method	System											
	Air–oil		Air–water		Steam–water		Refrigerants		Cryogenics		Overall	
	$\bar{X}_{\ln}$	$S_{\ln}$	$\bar{X}_{\ln}$	$S_{\ln}$	$\bar{X}_{\ln}$	$S_{\ln}$	$\bar{X}_{\ln}$	$S_{\ln}$	$\bar{X}_{\ln}$	$S_{\ln}$	$\bar{X}_{\ln}$	$S_{\ln}$
Stratified smooth flow												
Agrawal et al.	0.068	0.499	0.111	0.619	–	–	0.809	1.098	–	–	0.048	0.593
Baker and Gravestock	−0.813	6.221	−0.885	8.901	–	–	−0.593	1.879	–	–	−0.849	7.054
Hanratty	0.156	1.396	−0.31	2.464	–	–	1.585	2.122	–	–	−0.019	1.680
Hart et al.	−0.528	2.306	0.695	3.926	–	–	−0.037	0.459	–	–	−0.601	2.734
Hashizume et al.	0.145	0.561	0.452	1.361	–	–	0.733	1.064	–	–	0.193	0.937
Johannessen	1.372	1.748	0.936	1.173	–	–	3.32	4.296	–	–	1.125	1.448
Persen	1.886	3.207	0.771	1.917	–	–	4.282	5.831	–	–	1.347	2.454
Russel et al.	1.943	2.239	3.244	4.419	–	–	4.548	5.971	–	–	2.362	3.035
Taitel and Dukler	2.234	2.49	1.718	1.507	–	–	3.741	4.935	–	–	1.795	2.032
Taitel et al.	1.944	2.662	2.122	0.001	–	–	1.873	2.495	–	–	0.660	2.072
Stratified wavy flow												
Agrawal et al.	0.54	0.706	0.51	0.708	–	–	0.337	0.561	0.413	0.693	0.464	0.677
Baker and Gravestock	−0.25	0.471	−0.178	0.519	–	–	−0.235	0.574	−0.247	0.726	−0.191	0.532
Hanratty	−0.649	2.69	−0.734	4.907	–	–	−0.654	2.657	−0.567	2.027	−0.692	3.306
Hart et al.	0.096	0.464	0.18	0.577	–	–	0.373	0.568	0.425	0.93	0.161	0.585
Hashizume et al.	−0.017	0.394	0.139	0.51	–	–	−0.104	0.371	−0.219	1.272	0.039	0.541
Johannessen	0.38	0.675	0.752	0.955	–	–	0.452	0.579	0.558	0.856	0.605	0.822
Persen	−0.372	1.834	0.221	1.345	–	–	0.177	0.751	0.068	0.563	0.119	1.169
Russel et al.	1.238	1.475	1.623	2.24	–	–	1.063	1.232	1.936	2.188	1.366	1.831
Taitel and Dukler	1.092	1.277	1.107	1.328	–	–	0.704	0.805	1.108	1.36	0.936	1.175
Taitel et al.	1.162	1.433	0.907	1.118	–	–	0.603	0.864	0.636	1.116	0.892	1.157
Annular-dispersed flow												
Hart et al.	0.284	0.746	0.45	0.83	0.593	0.957	0.441	0.667	0.402	0.653	0.446	0.900
Hashizume et al.	−0.05	0.627	0.089	0.566	−0.090	−0.564	0.029	0.361	0.090	0.412	−0.043	0.538
Taitel et al.	1.524	2.629	0.287	0.781	−0.236	1.136	0.025	0.605	−0.143	0.889	0.084	1.151
Intermittent flow												
Dukler and Hubbard	0.398	1.012	0.091	0.535	0.567	1.457	0.535	0.939	0.268	0.618	0.155	0.785
Nicholson et al.	−0.053	0.376	−0.004	0.445	0.048	0.613	0.502	0.969	−0.092	0.536	−0.023	0.583
Taitel et al.	0.377	0.955	0.160	0.500	0.078	0.63	0.876	1.295	0.240	0.492	0.199	0.742
Dispersed-bubble flow												
Nicholson et al.	−0.050	0.324	0.078	0.11	0.084	0.142	–	–	–	–	0.028	0.202
Taitel et al.	0.114	0.607	0.099	0.126	0.088	0.138	–	–	–	–	0.096	0.335



Bandel (1973)

Agrawal *et al.* (1973)

Friedel (1979)

Hart *et al.* (1989)

Müller-Steinhagen & Heck (1986)

Hanratty (1987)

Olujic (1985)

Hashizume *et al.* (1985)

Fig. 2. PDFs for error (*R*-1) associated with the prediction of stratified wavy flow data. Empirical correlations (left column) and phenomenological models (right column).

Friedel (1979) method is notable, which is often quoted as the best of the simple, empirical methods for two-phase pressure drop (Whalley, 1982; Holt et al., 1997).

5.2. Stratified wavy flow

The most consistently accurate predictions are generated by the empirical methods of Bandel (1973) and Olujic (1985). Once again, errors associated with the Friedel (1979) correlation are spread over a wide range, as illustrated by the PDF in Fig. 2. PDFs for a number of other methods are also presented in Fig. 2. Error is defined using the ratio of measured to predicted values. The parameter R-1 defines the error such that perfectly accurate predictions, for which R is unity, have a value of zero on the error scale.

Table 3 shows that for stratified wavy flow the Lockhart and Martinelli (1949) correlation is more accurate for air–water and air–oil systems, reflecting the use of data from both systems in the method's construction. Data for other fluid systems are less well predicted. The Müller-Steinhagen and Heck (1986) correlation is also unable to provide generic accuracy with respect to changes in fluid system, as illustrated in Fig. 2.

Empirical methods still appear more able to reliably predict pressure gradients in this regime, indicating the difficulty in its modelling. However, the phenomenological models of Hanratty (1987) and Hart et al. (1989) are not significantly inferior even to the better empirical methods. The poor prediction of cryogenics flow data by both models appears to be linked to the definition of the flow pattern transition boundary. Cryogenics data are predicted by Taitel and Dukler (1976a) to be in the stratified wavy flow regime, while Hashizume (1983) indicates stratified smooth flow. The high scatter parameters observed in the statistical analysis appear more typical of those observed for stratified smooth flow data, and agree with Hashizume's flow pattern prediction.

The model by Agrawal et al. (1973) consistently tends towards underprediction. Although its performance compared to that in the stratified smooth regime is reduced by the presence of surface waves, it remains only a weak function of fluid system, as shown by the PDFs in Fig. 2. Qualitative analysis of the influence of interfacial structure on error revealed no apparent relationship between error and position on the flow pattern map.

The fact that models are capable of predictions as accurate as the best empirical methods is a clear statement of the potential of the approach. Further phenomenon specific data will surely result in future prediction performance in excess of that of empirical methods.

5.3. Stratified–intermittent flow transition

A region of poor precision exists at the transition boundary between stratified wavy flow, intermittent flow and annular flow. This is observed for a number of prediction methods, both empirical and phenomenological. Measured data in the region have a wide natural spread. Deterministic prediction methods are unable to account for such a variation.

It appears that two-phase phenomena, such as slug frequency and slug length, are stochastic in nature and are misrepresented by deterministic correlations. Thus, the difference between experimental and predicted values is a function of the experimental conditions present at the time of measurement. Unfortunately, information regarding such phenomena is absent from

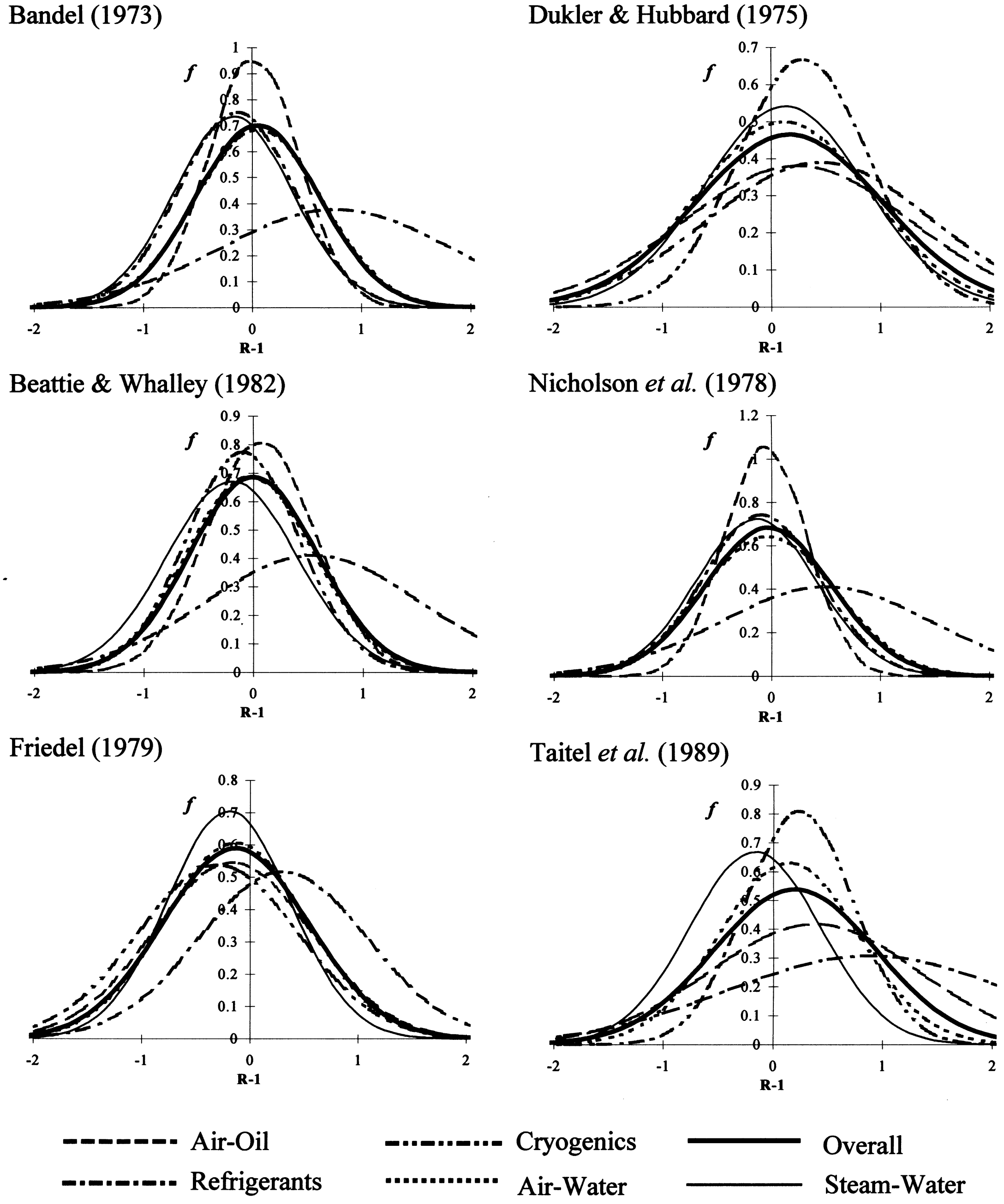


Fig. 3. PDFs for error (*R*-1) associated with the prediction of intermittent flow data. Empirical correlations (left column) and phenomenological models (right column).

the TVT-Dukler data bank; inclusion of such data should be seen as a priority in future two-phase data bank assembly.

5.4. Intermittent flow

Intermittent flow is neither uniform nor steady. Empirical methods are unable to completely define this pattern, as they only generate temporal and spatial averages.

In spite of this disadvantage, empirical methods have been traditionally employed. In this analysis, the methods of Bandel (1973) and Beattie and Whalley (1982) provided the best performance (Fig. 3). Although the latter is based on the homogeneous flow theory, its good performance reflects the domination, particularly at low flow qualities, of the frictional component due to the liquid slug. The slug is often modelled as a quasi-homogeneous two-phase flow, and the inclusion of this assumption in phenomenological models appears to be valid.

The Dukler and Hubbard (1975) model is solved using the slug frequency correlation of Grescovich and Shrier (1972), while the model of Nicholson et al. (1978) is solved assuming a slug length of 30 pipe diameters. It is found that the performance of the Nicholson et al. (1978) model is quite insensitive to this assumption, which vindicates its widespread use in intermittent flow modelling (e.g., de Henau and Raithby, 1995).

The model proposed by Nicholson et al. (1978) is found to be the most accurate of those analysed. Its precision is equal to that of the best empirical method (Bandel, 1973), while its simultaneous prediction of other parameters makes it considerably more powerful. Fig. 3 shows it to be vastly superior to the other two models in terms of precision, accuracy and system sensitivity. The poor prediction of refrigerant data is common to all methods. The intermittent flow data for refrigerants exists near the transition boundary with stratified wavy flow. As mentioned in the previous section, predictions made in this region tend to have high spread mainly due to the natural spread of the data.

5.5. Annular flow

Fig. 4 includes the PDFs for six methods. Again, the empirical methods show marked system sensitivity. Air–oil data are particularly poorly predicted. The Bandel (1973) correlation is the best performing empirical method.

System sensitivity is clearly evident with the phenomenological model of Taitel et al. (1989). However, the PDFs of Hashizume et al. (1985) are shown to be quite insensitive to changes in fluid system, and this model provides the most consistently accurate predictions of all methods tested in annular flow.

The PDFs of the Apparent Rough Surface model (ARS model) of Hart et al. (1989) in Fig. 4 show a large spread and a tendency towards underprediction. The model's intended field of application covers a small region which crosses the transition boundary between stratified and annular flows, and does not conform to any of the regions defined by the map of Taitel and Dukler (1976a). In this investigation, the model is purposefully used outside of the boundaries imposed by its authors, i.e. $0 < \varepsilon < 0.06$, with the aim of finding generic capabilities within the flow patterns defined by Taitel and Dukler (1976a). Further, qualitative examination of the results showed that the accuracy of the ARS model is severely reduced once the predicted wetted wall fraction reaches unity. Accurate predictions are made under incomplete wetting conditions. Analysing performance within a rigidly defined flow regime is inappropriate for this model and, similar to discussion in previous sections, it is evident that the use of these

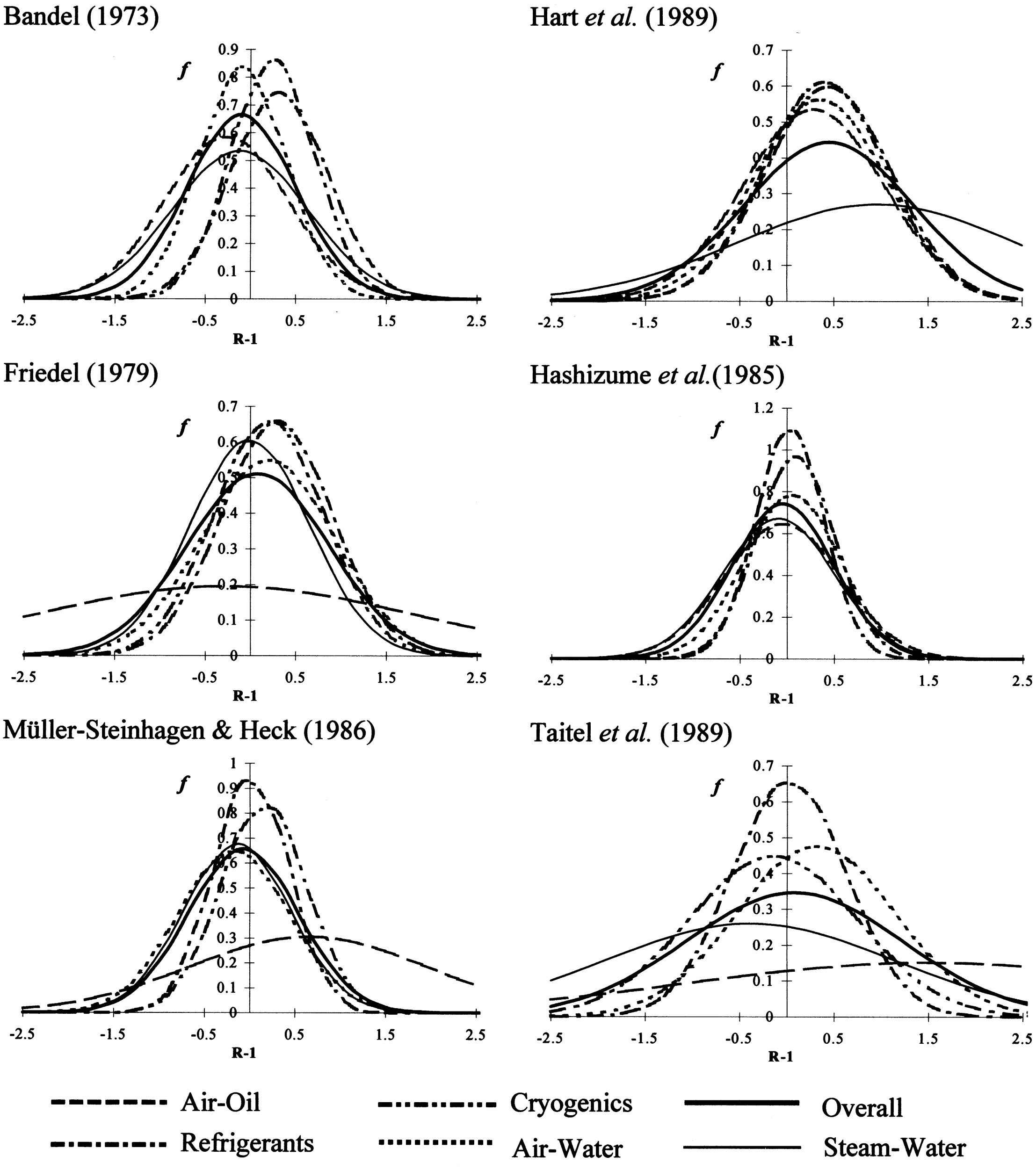


Fig. 4. PDFs for error (R-1) associated with the prediction of annular dispersed flow data. Empirical correlations (left column) and phenomenological models (right column).

boundaries is unduly affecting the error statistics and under-rating the capabilities of the ARS model. Qualitative analysis is shown to be an important part of the evaluation and subsequent selection process.

5.6. Dispersed bubble flow

Empirical methods developed specifically for this regime tend to treat it as a homogeneous flow. It is particularly interesting to note that the predictions of most empirical methods are extremely good for air–water data, but are significantly less accurate for other fluid systems. The preferred use of air–water data to formulate prediction methods is clearly evident. This is particularly well illustrated in Fig. 5 by the PDFs for the Beattie and Whalley (1982) correlation.

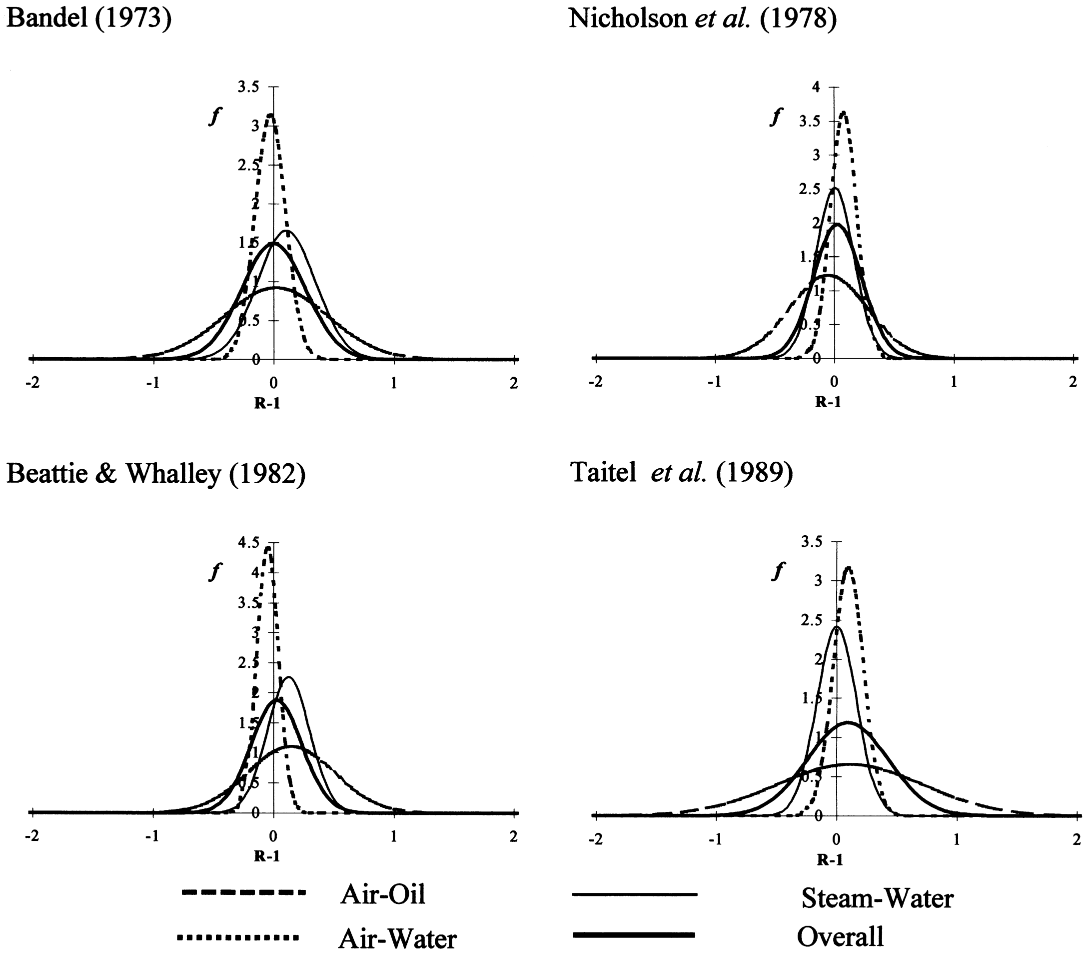


Fig. 5. PDFs for error (R-1) associated with the prediction of dispersed bubble flow data. Empirical correlations (left column) and phenomenological models (right column).

Table 4
Composite methods constructed to enhance prediction accuracy

Method	Stratified smooth	Stratified wavy	Annular dispersed	Intermittent	Dispersed bubble
1	Agrawal et al.	Hashizume et al.	Hashizume et al.	Nicholson et al.	Nicholson et al.
2	Agrawal et al.	Hart et al.[a]	Hashizume et al.[a]	Nicholson et al.[a]	Nicholson et al.[a]

[a] Taitel and Dukler (1976a) flow pattern map not applied to these flow pattern transitions.

The same is true of phenomenological models, again due to the predominant use of air–water phenomenon specific data. Nevertheless, the accuracy of prediction in this regime is better than for other patterns.

Nicholson et al. (1978) consider the transition from slug flow to bubbly flow to be the result of suppression of the Taylor bubble and enlargement of the quasi-homogeneous liquid slug. Taitel et al. (1989) treat the transition in quite the opposite manner. They assume that the Taylor bubble becomes the dominant gas-phase feature as the gas content in the liquid slug diminishes.

The statistically derived PDFs for the two methods may be compared in Fig. 3, where it can be seen that the Nicholson et al. (1978) model is superior. The assumption of homogeneity is vindicated.

Table 5
PDF parameters for generic prediction methods against all data

	System											
	Air–oil		Air–water		Steam–water		Refrigerants		Cryogenics		Overall	
Method	$\bar{X}_{\ln}$	$S_{\ln}$	$\bar{X}_{\ln}$	$S_{\ln}$	$\bar{X}_{\ln}$	$S_{\ln}$	$\bar{X}_{\ln}$	$S_{\ln}$	$\bar{X}_{\ln}$	$S_{\ln}$	$\bar{X}_{\ln}$	$S_{\ln}$
Empirical methods												
Bandel	−0.087	0.491	−0.016	0.491	−0.118	0.724	0.333	0.596	0.134	0.474	−0.044	0.566
Beattie & Whalley	0.062	0.766	0.014	0.596	−0.083	0.693	0.275	0.662	0.211	0.585	0.048	0.657
Friedel	−0.225	1.298	−0.015	0.729	−0.020	0.645	0.154	0.656	−0.001	0.654	−0.035	0.784
Lockhart & Martinelli	0.096	0.426	−0.016	0.482	−0.553	1.167	−0.134	0.625	−0.368	1.196	−0.162	0.836
Müller-Steinhagen	0.414	0.867	−0.124	0.693	−0.130	0.576	−0.028	0.597	0.036	0.488	−0.044	0.658
Olujic	0.451	1.168	0.018	0.461	−0.313	1.297	0.073	0.418	0.061		0.464	0.710
Composite methods												
Mukherjee & Brill	0.016	1.265	−0.222	1.285	−0.228	0.851	−0.226	1.288	−0.373	0.961	−0.176	1.179
Hashizume et al.	0.467	1.23	0.324	0.897	−0.023	0.635	0.151	0.586	0.147	0.935	0.205	1.113
Taitel et al.	0.934	1.813	0.346	0.850	−0.352	1.542	0.219	0.780	0.019	0.918	0.172	0.837
Method 1	−0.042	0.453	0.045	0.531	−0.095	0.587	0.048	0.443	−0.006	0.617	−0.028	0.540
Method 2	0.03	0.533	0.044	0.497	−0.076	0.573	0.102	0.466	0.059	0.505	0.012	0.514

5.7. Composite models

Individual models acting alone have been shown to offer accurate predictions within their designated flow patterns. To enable prediction across a range of conditions, a number of models must be used under the control of a flow pattern map.

The flow pattern based analyses discussed in the previous paragraphs have lead to the recommendation of two composite methods constructed only from phenomenological models. Models were selected on merit, and are listed in Table 4.

Method 1 is based entirely on the flow pattern based statistical results obtained using the Taitel and Dukler (1976a) flow pattern map. Thus, the effect on the statistical parameters associated with the position of the flow pattern transition boundary is not taken into consideration.

Method 2 includes qualitative observations made by plotting error as a function of position in the flow pattern map. This resulted in the Hart et al. (1989) model being specified for use

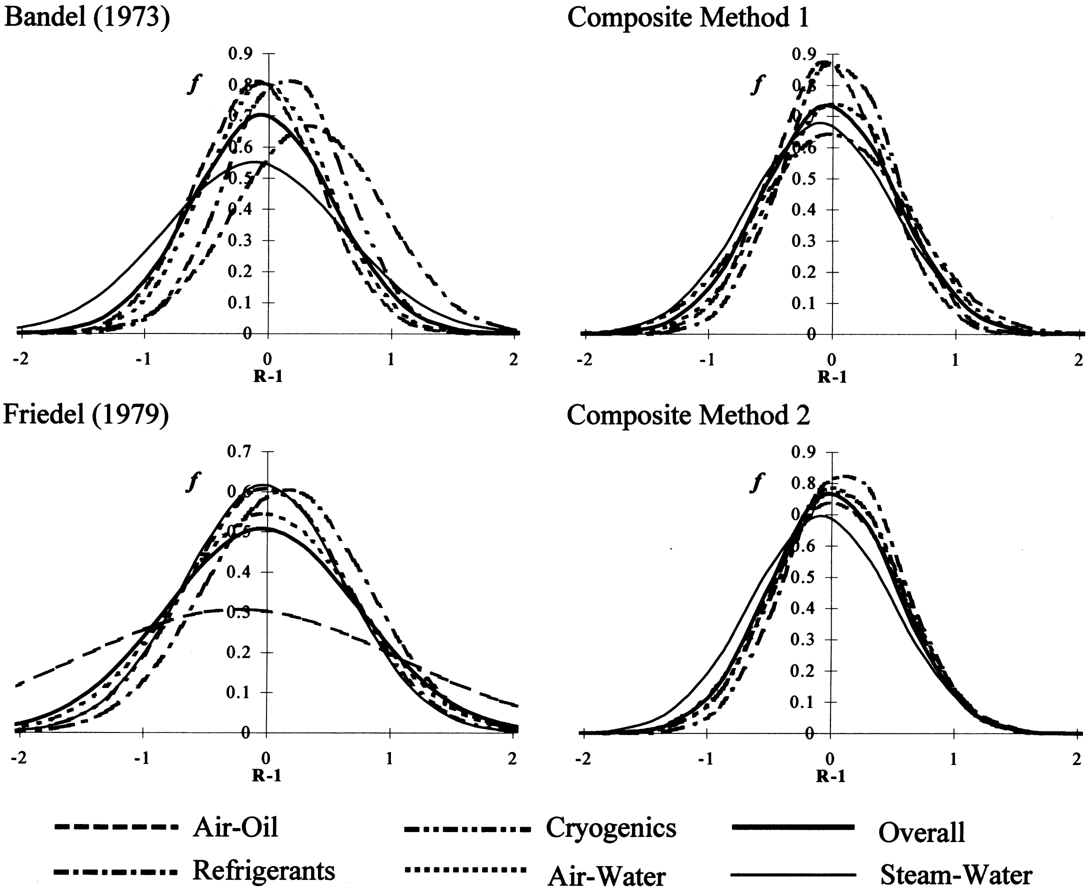


Fig. 6. PDFs for error (*R*-1) associated with prediction of data for various fluid systems by generic methods.

in stratified wavy flow up to the point at which the wetted wall fraction becomes unity. Beyond that, the annular-dispersed flow model of Hashizume et al. (1985) is preferred. The transition from intermittent flow to dispersed-bubble flow is defined by Nicholson et al. (1978).

The variation between the empirical and phenomenological approaches as generic prediction methods is best seen in this section as the effects of flow pattern transition are included implicitly within the framework of the statistical analysis.

According to the statistical evidence in Table 5, the best stand-alone empirical method is that proposed by Bandel (1973). Fig. 6 shows this method's performance to be superior to that of the Friedel (1979) correlation, although both show inconsistency due to fluid system.

Composite methods found in the literature have not performed well as empirical methods. Assumptions made in the methods of Taitel et al. (1989) and Mukherjee and Brill (1985) are overly simple, while the method proposed by Hashizume et al. (1985) is incomplete and requires additional models for intermittent flow and dispersed bubble flow.

However, the composite methods derived from this study are seen to provide good prediction accuracy with a minimum of sensitivity to fluid system (Fig. 6). Method 2 is the more accurate method, and reduces the standard deviation of the overall distribution by 10% compared to the best empirical correlation (Bandel, 1973). The statistical mean value is also reduced. Most remarkable, however, is the reduction in fluid system sensitivity.

6. Conclusions

Previous performance evaluations of pressure drop prediction techniques have focused on empirical methods only. This paper outlines an investigation into the performance of phenomenological models. Data are categorised into flow patterns using the Taitel and Dukler (1976a) flow pattern map. A logarithmic analysis technique is used to define the parameters describing the error distributions.

The performance of phenomenological models is shown to be as good as that of empirical methods. They provide additional information regarding interfacial structure making them the preferred choice.

It has been shown that phenomenological methods are less influenced by changes of fluid system. Therefore, such models are potentially more capable of accurate predictions in systems for which data are not available in data banks.

Flow pattern transition boundaries are shown to be regions of poor prediction precision. The suspected stochastic nature of two-phase phenomena, particularly during intermittent flow, cause natural variations in measured pressure drop data. Predictions made using deterministic correlations are incapable of reflecting this distribution of pressure drop values.

This region of poor performance often influences the statistical results indicative of a method's performance. Thus, error is potentially a function of the position on the flow pattern map. This is shown to be true in a number of cases, particularly for phenomenological models. Likewise, PDFs are also affected by the position of the transition boundaries. The value in assessing the sensitivity to transition boundaries by a qualitative analysis is shown.

Composite methods made up of a number of phenomenological models are recommended. Method 1 selects those models which indicate best performance from the statistical analysis. Method 2 includes qualitative information regarding the relationship between error and the flow pattern map.

Both composite methods clearly outperform the best empirical methods, with Method 2 reducing the scatter by 10% as compared to Bandel's (Bandel, 1973) correlation.

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